Numbers, Functions, Equations

Hotel Aurum, Hajdúszoboszló (Hungary)

August 26 – September 1, 2018

FNE 2018

Institute of Mathematics
University of Debrecen

The conference is dedicated to

the 80th birthday of Professor	**Zoltán Daróczy**
the 80th birthday of Professor	**Imre Kátai**
the 75th birthday of Professor	**Karl-Heinz Indlekofer**
the 70th birthday of Professor	**Jean-Marie De Koninck**
the 70th birthday of Professor	**Gyula Maksa**
the 65th birthday of Professor	**Bui Minh Phong**
the 60th birthday of Professor	**Sándor Fridli**

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Amr Zakaria	Debrecen,	Hungary	

General Information

The conference is held in Hotel Aurum in Hajdúszoboszló, Hungary, from Sunday, August 26 (arrival day) to Saturday, September 1, 2018 (departure day). The participation fee covers full board, accommodation (in double rooms), the use of the spa and the sauna in Hotel Aurum, registration materials, the fees of the excursion, the banquet and other social events of the meeting. The participants are put up in Hotel Aurum.

All conference *talks* are given in the *Main Lecture Room*, which is equipped with a computer, a data projector, and a whiteboard. The Main Lecture Room and the Coffee Breaks are in the In Hotel. The duration of every regular talk is at most 20 minutes, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk. The time saved by shorter talks can be devoted to problems and remarks at the end of the session. If you have special wishes concerning the schedule, you are welcome to consult Dr. Mihály Bessenyei.

During the conference, you have the possibility to photocopy and print a limited number of pages in the Conference Office. Internet is also available in the hotel with a wireless connection.

You can find the program, the list of participants, the list of talks and the abstracts in this booklet. The schedule of the individual sessions and further technical details are announced during the conference. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact us. We hope that our conference will be interesting and successful and you will enjoy your stay in Hajdúszoboszló.

– Program –

Monday		Tuesday	
07:00–09:00	Breakfast	07:00–09:00	Breakfast
09:00–10:40	1st Morning Session	09:00–10:40	1st Morning Session
10:40–11:00	Coffee Break	10:40–11:00	Coffee Break
11:00–12:40	2nd Morning Session	11:00–12:40	2nd Morning Session
12:45–14:30	Lunch	12:45–14:30	Lunch
15:30–17:10	1st Afternoon Session	15:30–17:10	1st Afternoon Session
17:10–17:30	Coffee Break	17:10–17:30	Coffee Break
17:30–19:10	2nd Afternoon Session	17:30–19:10	2nd Afternoon Session
19:15–21:00	Dinner	19:15–21:00	Dinner

Wednesday		Thursday	
07:00–09:00	Breakfast	07:00–09:00	Breakfast
09:00–10:15	1st Morning Session	09:00–10:40	Laudatio
10:15–10:35	Coffee Break	10:40–11:00	Coffee Break
10:35–11:50	2nd Morning Session	11:00–12:30	Laudatio
12:00–13:00	Lunch	12:45–14:30	Lunch
13:00–	Excursion to Tokaj	15:00–16:00	Laudatio
	Dinner in Rátka	18:00–22:00	Festive Dinner

Friday		Saturday	
07:00–09:00	Breakfast	07:00–09:00	Breakfast
09:00–10:40	1st Morning Session		
10:40–11:00	Coffee Break		
11:00–12:40	2nd Morning Session		
12:45–14:30	Lunch		
15:30–17:10	1st Afternoon Session		
17:10–17:30	Coffee Break		
17:30–19:10	2nd Afternoon Session		
19:15–21:00	Dinner		

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– List of Talks –

1. **Gintautas Bareikis**, *Beta distribution on arithmetical semigroups*
2. **Karol Baron**, *Weak limit of iterates of random-valued functions and solvability of a linear iterative equation*
3. **Mihály Bessenyei**, *Generalized monotonicity in terms of differential inequalities*
4. **István Blahota**, *Approximation by Theta-means*
5. **Gergő Bognár**, *Geometric interpretation of QRS complexes in ECG signals by rational functions*
6. **Zoltán Boros**, *Conditionally linked monomial functions*
7. **Pál Burai**, *On symmetry of Makó–Páles means*
8. **Péter Burcsi**, *Laudation to Bui Minh Phong*
9. **Péter Burcsi**, *Laudation to Jean-Marie De Koninck*
10. **Péter Burcsi**, *Some algorithmic questions in numeration systems*
11. **Jacek Chudziak**, *Buying and selling prices under the Rank-Dependent Utility model*
12. **Bálint Daróczy**, *Riemann manifolds on hierarchical structures*
13. **Nicolas Doyon**, *A transition theorem for connected feedforward graphs*
14. **Gábor Farkas**, *Laudation to Imre Kátai*
15. **Borbála Fazekas**, *Walsh–Fourier series solutions of linear differential equations with error estimation*
16. **Włodzimierz Fechner**, *On a Grüss-type functional inequality*
17. **Żywilla Fechner**, *Some remarks on a moment functions problem*
18. **Ágnes Fülöp**, *Interdisciplinarity between the informatics and physics*
19. **György Gát**, *Summability of trigonometric and Walsh–Fourier series*
20. **Roman Ger**, *Solving alternative functional equations: what for?*
21. **Attila Gilányi**, *Solving functional equations with computer*
22. **Dorota Głazowska**, *On weighted quasi-arithmetic means which are convex*
23. **Eszter Gselmann**, *Laudation to Gyula Maksa*
24. **Bui Xuan Hai**, *Solution of one conjecture on POS-groups*
25. **László Horváth**, *Sharp estimation for the solutions of inhomogeneous delay differential and Halanay-type inequalities*
26. **Witold Jarczyk**, *Continuous solutions of some simultaneous difference equations on a restricted domain*
27. **Roma Kačinskaitė**, *Joint discrete mixed universality for certain zeta- and L-functions*
28. **Gábor Kallós**, *Beta expansions and dynamical systems*
29. **Tibor Kiss**, *On two functional equations related to equality problem of means*
30. **Oleg Klesov**, *Dual objects in function theory, probability theory, and number theory*
31. **Oleksiy Klurman**, *Joint distribution of values of multiplicative functions*
32. **Attila Kovács**, *Laudation to Karl-Heinz Indlekofer*
33. **Attila Kovács**, *On lattice based number systems*

34. **Miklós Laczkovich**, *Continuous solutions of the equation* $x + g(y + f(x)) = y + g(x + f(y))$
35. **Antanas Laurinčikas**, *Joint value distribution theorems for the Riemann and Hurwitz zeta functions*
36. **László Losonczi**, *On the zeros of reciprocal polynomials*
37. **Levente Lócsi**, *The Blaschke group and rational Zernike functions*
38. **Eugenijus Manstavičius**, *Probabilistic number theory on permutations*
39. **Gergő Nagy**, *Transformations on positive operators with preserver properties related to means*
40. **Károly Nagy**, *Cone-like restricted summability of the two-dimensional Walsh- and Walsh–Kaczmarz means*
41. **Gábor Nagy**, *On non-integer based expansions of real numbers in a special form*
42. **Zsolt Németh**, *Projection properties of de la Vallée Poussin type operators*
43. **Vincent Ouellet**, *On the middle prime factors of integers*
44. **Zsolt Páles**, *Equality of Bajraktarević means with quasi-arithmetic means*
45. **Paweł Pasteczka**, *Jensen-type geometric shapes*
46. **Štefan Porubský**, *Semigroup structure of sets of solutions to equation* $X^m = X^s$
47. **Gábor Román**, *On the expected number of curve orders during the Atkin–Morain primality test*
48. **Lajos Rónyai**, *Recent results on norm-graphs*
49. **Maciej Sablik**, *An elementary method of solving functional equations*
50. **Peter Stadler**, *The short ruler on the real projective space*
51. **Gediminas Stepanauskas**, *Asymptotical behaviour of arithmetic functions on shifted primes*
52. **Kristóf Szarvas**, *The boundedness of the Cesaro means in variable dyadic Hardy spaces*
53. **László Székelyhidi**, *Laudation to Zoltán Daróczy*
54. **Patrícia Szokol**, *Transformations preserving generalized quasi-arithmetic means of invertible positive operators*
55. **Rodolfo Toledo**, *Numerical solution of linear differential equations by Walsh polynomials approach*
56. **Pavel Varbanets**, *Exponential sums over the sequences of PRN's produced by inversive generators*
57. **Peter Volkmann**, *An application of a comparison theorem for functional equations*
58. **Ferenc Weisz**, *Laudation to Sándor Fridli*
59. **Ferenc Weisz**, *Variable Hardy spaces and applications in Fourier analysis*
60. **Amr Zakaria**, *Invariance of a symmetric Bajraktarević mean with respect to two nonsymmetric Bajraktarević means*

Abstracts

Gintautas Bareikis
(Vilnius University)

Beta distribution on arithmetical semigroups

(joint work with Algirdas Mačiulis)

The sequences of the distributions defined on an arithmetical semigroup are considered. We prove that any Beta distribution can occur as a limit law for such sequences. Partial solution of this problem was obtained in [1].

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[1] G. Bareikis and A. Mačiulis, *On the numbers of divisors in arithmetical semigroup*, Ann. Univ. Sci. Budapest. Sect. Comp. **39** (2013), 35–44.

Karol Baron
(Silesian University of Katowice)

Weak limit of iterates of random-valued functions and solvability of a linear iterative equation

Given a probability space $(\Omega, \mathcal{A}, P)$, a complete and separable metric space X with the σ–algebra $\mathcal{B}$ of all its Borel subsets, a $\mathcal{B} \otimes \mathcal{A}$–measurable $f : X \times \Omega \to X$ and a bounded and Lipschitz F mapping X into a separable Banach space Y we consider the problem of the solvability of the equation

$$\varphi(x) = \int_{\Omega} \varphi\left(f(x,\omega)\right) P(d\omega) + F(x)$$

in the class of Lipschitz functions $\varphi : X \to Y$ and characterize it with the aid of the weak limit of the sequence of iterates $\left(f^n(x,\cdot)\right)_{n\in\mathbb{N}}$ defined as in [1, Section 1.4].

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[1] M. Kuczma, B. Choczewski and R. Ger, *Iterative functional equations*, Encyclopedia of mathematics and its applications, Vol. 32, Cambridge University Press, Cambridge, 1990.

Mihály Bessenyei
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Generalized monotonicity in terms of differential inequalities

The classical notions of monotonicity and convexity can be extended applying Chebyshev systems [2]. The main result of the talk [1], analogously to the classical derivative tests, characterizes generalized monotonicity in terms of differential inequalities. Applications in the fields of convexity and ordinary differential equations are also discussed. The key tool of the approach is the Páles mean-value theorem [3].

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[1] M. Bessenyei, *Generalized monotonicity in terms of differential inequalities*, Proc. Roy. Soc. Edinburgh Sect. A (2018), to appear.
[2] S. Karlin and W. J. Studden, *Tchebycheff systems: With applications in analysis and statistics*, Pure and Applied Mathematics, Vol. XV, Interscience Publishers John Wiley & Sons, New York-London-Sydney, 1966.
[3] Zs. Páles, *A general mean value theorem*, Publ. Math. Debrecen **89** (2016), 161–172.

István Blahota

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Approximation by Θ-means

(joint work with Károly Nagy)

The topic of this lecture is Θ-means of (one- and two-dimensional) Walsh series of a function in L^p and in $\mathrm{Lip}(\alpha, p)$, where $\alpha > 0$ and $1 \le p \le \infty$.

Our theorems give generalizations (and two-dimensional analogues) of results of Móricz, Siddiqi on Nörlund means [3], Móricz, Rhoades on weighted means [2] and Fridli, Manchanda and Siddiqi [1] for homogeneous Banach spaces.

Our two-dimensional theorem generalizes two results of Nagy on Nörlund means and weighted means of the cubical partial sums of double Walsh–Fourier series [4, 5].

References

[1] S. Fridli, P. Manchanda and A. H. Siddiqi, *Approximation by Walsh–Nörlund means*, Acta Sci. Math. (Szeged) **74** (2008), 593–608.

[2] F. Móricz and B. E. Rhoades, *Approximation by weighted means of Walsh-Fourier series*, Int. J. Math. Sci. **19** (1996), 1–8.

[3] F. Móricz and A. Siddiqi, *Approximation by Nörlund means of Walsh–Fourier series*, J. Approx. Theory **70** (1992), 375–389.

[4] K. Nagy, *Approximation by Nörlund means of quadratical partial sums of double Walsh–Fourier series*, Anal. Math. **36** (2010), 299–319.

[5] K. Nagy, *Approximation by weighted means of cubical partial sums of double Walsh–Fourier series*, Jaen J. Approx. **2** (2010), 147–161.

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Geometric interpretation of QRS complexes in ECG signals by rational functions

(joint work with Sándor Fridli, Péter Kovács and Ferenc Schipp)

Our subject is an adaptive transformation technique we have developed [1], inspired by problems in ECG signal processing, including heartbeat modelling [2], classification [3], compression [4], and medical parameter extraction. The transformation is a variable projection method involving rational function systems. The most important property of this method is that we have a large pool of systems at hand, that can be adjusted to the individual signal. As a result not only the coefficients of the rational projection but also the system parameters carry information about the signal.

At first, we give a summary of our results, with the focus on the mathematical challenges we have encountered, depending on the specific problems. We discuss among others the orthogonal and biorthogonal representations of the model; the identification of the system parameters which is a nonlinear optimization problem; the approximation and representation abilities.

Then we present our latest research on the mathematical modelling for QRS complexes of ECG heartbeats with rational functions. We investigated the geometric properties of the QRS complexes based on this model. Among others, we explored the connection between the fiducial points and parameters of the rational transform, via the roots and the local extrema of the model curve. The importance of this model is twofold. On the first hand, the model provides an analytic way to determine medical descriptors of the QRS complex. On the other hand, it makes possible to synthesize heartbeats from an analytic model that fits the given descriptors.

References

[1] S. Fridli, L. Lócsi, and F. Schipp, *Rational function systems in ECG processing*, in Computer Aided Systems Theory – EUROCAST 2011: 13th International Conference, Las Palmas de Gran Canaria, Spain, February 6-11, 2011, Revised Selected Papers, Part I, LNCS 6927, R. Moreno-Díaz et al., Eds. Springer-Verlag Berlin Heidelberg, (2012), 88–95.

[2] S. Fridli, P. Kovács, L. Lócsi, and F. Schipp, *Rational modeling of multi-lead QRS complexes in ECG signals*, Ann. Univ. Sci. Budapest. Sect. Comp. **37** (2012), 145–155.

[3] G. Bognár and S. Fridli, *Heartbeat classification of ECG signals using rational function systems*, in Computer Aided Systems Theory – EUROCAST 2017: 16th International Conference, Las Palmas de Gran Canaria, Spain, February 19-24, 2017, Revised Selected Papers, Part II, LNCS 10672, R. Moreno-Díaz et al., Eds. Springer, Cham (2018), 187–195.

[4] P. Kovács, S. Fridli and F. Schipp, *Rational Generalized Variable Projection with Application in ECG processing* (2018), to appear.

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Conditionally linked monomial functions

(joint work with Edit Garda-Mátyás)

We establish the following result.

Theorem. *Let* $a\,,\,b \in \mathbb{R}$ *and let* $f\,,\,g\colon \mathbb{R} \to \mathbb{R}$ *be generalized monomials of the same degree such that* $g(b) \neq 0$. *If*

$$f(x)g(ax+b) = g(x)f(ax+b)$$

holds for every $x \in \mathbb{R}$, *then there exists* $c \in \mathbb{R}$ *such that* $f(x) = cg(x)$ *for all* $x \in \mathbb{R}$.

Further similar problems, statements and counterexamples are presented as well. This research was initiated in a recent paper on generalized monomials of degree two [1].

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Pál Burai
(University of Debrecen)

On symmetry of Makó-Páles means

(joint work with Justyna Jarczyk)

Given a nonempty real interval I, a continuous, strictly monotonic function $\varphi\colon I \to \mathbb{R}$, and a Borel probability measure μ on $[0,1]$, we characterize all symmetric Makó-Páles means $M_{\varphi,\mu}$ defined on $I \times I$ by

$$M_{\varphi,\mu}(x,y) = \varphi^{-1}\left(\int_0^1 \varphi(tx+(1-t)y)d\mu(t)\right).$$

Jacek Chudziak
(University of Rzeszów)

Buying and selling prices under the Rank-Dependent Utility model

Assume that $\mathcal{X}$ is a family of all *risky lotteries*, that is finitely supported probability distributions on $\mathbb{R}$. Let $u : \mathbb{R} \to \mathbb{R}$ be a strictly increasing continuous utility function. Under the Expected Utility model, *a buying price* $B(W, X)$ and *a selling price* $S(W, X)$ for a lottery $X \in \mathcal{X}$ at a decision maker's wealth level $W \in \mathbb{R}$ is defined as a unique real number satisfying the equation

$$E[u(W + X - B(W, X))] = u(W)$$

and

$$E[u(W + X)] = u(W + S(W, X)),$$

respectively. Several results concerning the buying and selling prices under the Expected Utility model can be found in [1]. In the talk we present some properties of the prices under the empirically motivated Rank Dependent Utility model.

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Bálint Daróczy
(Hungarian Academy of Sciences)

Riemann manifolds on hierarchical structures

Hierarchical neural networks are exponentially more efficient than their corresponding "shallow" counterpart with the same expressive power, but involve huge number of parameters and require tedious amounts of training. We consider the case of low order polynomials as Hamiltonians following the results in [1]. The expressive power of deep structures with a special family of metrics [2, 3] suggest us sparse representations and various metrics. We investigate how the quality of approximation connected to the geometrical properties of the learning methods in certain hierarchical, deep structures without efficient "flattening".

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Nicolas Doyon
(University of Laval)

A variation of Erdős-Rényi transition Theorem for connected feedforward graphs

(joint work with Anthony Richard)

Erdős and Rényi have shown in their classical work that the probability that a random simple graph with n nodes is connected exhibits a sharp transition when the probability of edge existence reaches $\log n/n$. We obtain similar results for directed and feedforward graphs. Using Pólya Theorem, we also obtain exact and asymptotic enumeration theorems for classes of such isomorphic graphs.

Borbála Fazekas
(University of Debrecen)

Walsh–Fourier series solutions of linear differential equations with error estimation

Walsh functions $\omega_0, \omega_1, \dots$ are step functions with range $\{-1, 1\}$ defined on the interval $[0, 1[$. More precisely they are given via

$$\omega_n(x) = \prod_{k=0}^{\infty} (-1)^{n_k x_k}, \quad \text{with } n \in \mathbb{N}, n = \sum_{k=0}^{\infty} n_k 2^k, x \in [0,1[,\ x = \sum_{k=0}^{\infty} \frac{x_k}{2^{k+1}}.$$

The n-th Walsh-Fourier series approximation of an integrable function u on $[0, 1[$ is defined by

$$\sum_{k=0}^{\infty} \hat{u}(k)\omega_k(x), \quad \text{with } \hat{u}(k) = \int_0^1 u(t)\omega_k(t)\, dt.$$

One can look for an approximate solution of an ordinary differential equation in the above form by solving a suitably discretised equation. We apply this approach to the linear differential equation

$$y'(x) + f \cdot y(x) = g(x).$$

We give an upper bound for the error term $|y(x) - \hat{y}_n(x)|$, where $x \in [0, 1[$ and y denotes the exact solution and $\hat{y}_n$ the n-th approximate solution of the equation, respectively.

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Włodzimierz Fechner
(Łódź University of Technology)

On a Grüss type functional inequality

Let $f, g\colon [a,b] \to \mathbb{R}$ be integrable functions on a closed interval $[a,b]$. Assume that there exist some constants $\phi, \Phi, \gamma, \Gamma \in \mathbb{R}$ such that

$$\phi \leq f(x) \leq \Phi, \;\; \gamma \leq g(x) \leq \Gamma, \;\; x \in [a,b].$$

Chebyshev functional is defined as

$$T(f,g) := \frac{1}{b-a}\int_a^b f(x)g(x)\,\mathrm{d}x - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x \cdot \frac{1}{b-a}\int_a^b g(x)\,\mathrm{d}x.$$

Grüss inequality [2] says that

$$|T(f,g)| \leq \frac{1}{4}(\Phi - \phi)(\Gamma - \gamma).$$

Several generalizations and variants of Grüss inequality are well known. Dragomir [1] considered an arbitrary inner product space $(X, \langle \cdot, \cdot \rangle)$ and provided estimates for the value

$$|\langle f, g\rangle - \langle f, h\rangle \langle h, g\rangle|, \tag{1}$$

where $f, g, h \in X$, $\|h\| = 1$ and some bounds for f and g are assumed. Our purpose is to study functional inequalities stemming from known estimates of (1) and related results.

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Żywilla Fechner

(Silesian University of Katowice)

Some remarks on a moment functions problem

(joint work with László Székelyhidi)

We are going to discuss some problems concerning moment functions on hypergroups. Let $(K, e, ^\vee, *)$ be a locally compact hypergroup, i.e. a measure algebra over a locally compact Hausdorff space K with convolution satisfying certain axioms. Formal definition can be found e.g. in [2].

For any nonnegative integer N a continuous function $\varphi\colon K \to \mathbb{C}$ is called a *generalized moment function of order N*, if there exist complex valued continuous functions $\varphi_k\colon K \to \mathbb{C}$ such that $\varphi_N = \varphi$ and

$$\int_K \varphi_k(t)d(\delta_x * \delta_y)(t) = \sum_{j=0}^{k} \binom{k}{j} \varphi_j(x)\varphi_{k-j}(y) \tag{1}$$

holds for all $k = 0, 1, \ldots, N$ and for all $x, y \in K$. The functions $(\varphi_k)_{k\in\{0,1,\ldots,N\}}$ form a *generalized moment function sequence of order N*. Observe that φ_0 is an exponential on a hypergroup K. We say that f_0 *generates generalized moment function sequence of order N* and that the generalized moment functions in this sequence *correspond to* φ_0.

Our aim is to discuss motivation of these notions and give descriptions on different type of hypergroups.

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Ágnes Fülöp

(Eötvös Loránd University)

Interdisciplinarity between the infromatics and physics

The time sequences of measurements between chaotic and nonchaotic regions [1] are described by statistical complexity $C(H, D)$ [2] [3], which depends on the entropy H and the disequilibrium D on a finite N-system of the manifold of trajectories to apply the probability distribution of the finite measured systems.

Although the Kolmogorov-Sinai entropy [4] provides the tendency of the flow [5], the complexity C allows to determine the localisation of the strange attractor, the transition states and periodic motion in the parameter space near to the intermittent region considering the ergodicity.

The snapshot attractors can be applied to study the model of global atmospheric circulation [6] [7] using a driving system. We researched the chaoticity of this model by numerical approximation to analyse the statistical complexity of the time dependent attractor.

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György Gát

(University of Debrecen)

Summability of trigonometric and Walsh–Fourier series

In this talk a short résumé is given with respect to some recent results in the theory of summation of trigonometric and Walsh-Fourier series. An endpoint result regarding strong summation is also mentioned. We also discuss the case when only a subsequence of the sequence of partial sums is given. Finally, some problems and conjectures are formulated with respect to this issue.

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Roman Ger
(Silesian University of Katowice)

Solving alternative functional equations: what for?

To make it less abstract, answering the title question let us twist British mountaineer George Mallory's famous dictum: "Because they're there." The article yields a counterpart of Mihály Bessenyei and Gréta Szabó's paper published in Math. Mag. 91 (2018), 37-41, dealing with addition formula characterizing the tanh function. The emphasis is given on proving that a natural understanding of a solution of a functional in question requires to view that equation as an alternative one. For the most part the paper has a survey character.

Attila Gilányi
(University of Debrecen)

Solving functional equations with computer

(joint work with Gergő Gyula Borus)

In this talk, we present a program package developed in the computer algebra system Maple, which determines the solutions of two-variable linear functional equations.

Dorota Głazowska
(University of Zielona Góra)

On weighted quasi-arithmetic means which are convex

(joint work with Jacek Chudziak, Justyna Jarczyk and Witold Jarczyk)

We study convexity in the class of weighted quasi-arithmetic means. It turns out that their convexity depends only on the generator, neither on weights, nor on the number of variables. Connections between the convexity of a mean and the convexity of its increasing generators are considered. We prove that convex means are generated by convex strictly increasing functions. A simple example shows that the converse is not true, so the problem arises when this is the case. Some answers are given under regularity assumptions imposed on the generator.

Bui Xuan Hai
(VNUHCM-University of Science)

Solution of one conjecture on POS-groups

Let G be a finite group. For an element $x \in G$, the order of x is denoted by $o(x)$. We define the following equivalence relation in G:

$$x \sim y \Longleftrightarrow o(x) = o(y).$$

The equivalence class defined by an element x is denoted by $\overline{x}$ and is called *an order subset* of G. We say that G is a *group with perfect order subsets* or briefly, G is a POS-*group* if the number of elements in each order subset of G is a divisor of $|G|$. It turns out that every POS-group has even order. In 2002, Carrie E. Finch and Lenny Jones [2] studied abelian POS-groups and they introduced the notion of the so called *minimal* POS-*groups*. In fact, a minimal POS-group is such a POS-group G of the form $G \cong (\mathbb{Z}_2)^t \times M$, where $|M|$ is odd, and there is no a proper subgroup $\widehat{M}$ of M such that $(\mathbb{Z}_2)^t \times \widehat{M}$ is a POS-group. Carrie Finch and Lenny Jones proved that if $G \cong (\mathbb{Z}_2)^t \times M$ is a minimal abelian POS-group with $|M|$ square free, then the factor M is uniquely determined by the value t. Moreover, they gave the list of all POS-groups with this property. It is surprising that in the proof of this result, the fact that the Fermat number F_5 is composite is used as the essential argument. Since the symmetric group S_3 is a POS-group, Carrie Finch and Lenny Jones asked if there are non-abelian groups other than S_3 that are POS? In their further work [3], they proved that there exist infinitely many non-abelian POS-groups. Also, they posed the following conjecture: *For* $n \geq 4, A_n$ *and* S_n *are not* POS-*groups*. This conjecture is solved in the affirmative for A_n by A. K. Das in [1], and for S_n by N. T. Tuan and B. X. Hai in [4]. In the solution of this conjecture for the case of S_n, we only use the techniques of the elementary number theory. In particular, we use the Bertrand-Chebyshev theorem proved by Chebyshev in 1852. Later, in [5], Lenny Jones and Kelly Toppin provided another proof of the fact that $S_n, n \geq 4$ is not POS-group. Whereas, we use the Bertrand-Chebyshev theorem, they use the theorem proved by Nagura in 1952.

The aim of this talk is to demonstrate our solution of the problem above for the group $S_n, n \geq 4$ by using essentially the tools of the Elementary Number Theory.

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László Horváth
(University of Pannonia)

Sharp estimation for the solutions of inhomogeneous delay differential and Halanay-type inequalities

We consider inhomogeneous Halanay-type inequalities together with inhomogeneous linear delay differential inequalities and equations. Based on the the variation of constants formula and some results borrowed from a recent paper, sharp conditions for the boundedness and the existence of the limit of the nonnegative solutions are established. The sharpness of the results are illustrated by examples and by comparison of results in some earlier works.

Witold Jarczyk
(University of Zielona Góra)

Continuous solutions of some simultaneous difference equations on a restricted domain

(joint work with Paweł Pasteczka)

This is a work reporting a progress in studying the simultaneous equations

$$\varphi(tx) = \varphi(x) + c(t)x^p, \qquad t \in T,$$

where a set $T \subset (0, +\infty)$, a function $c\colon T \to \mathbb{R}$ and a real number p are given. The equations are postulated only for those $t \in T$ and $x \in I$ that $tx \in I$. We focus on an extension result allowing to obtain the form of continuous solutions on the whole interval I which up to now was known only on a subinterval of I.

Roma Kačinskaitė
(Vytautas Magnus University)

Joint discrete mixed universality for certain zeta- and L-functions

The first results related to the mixed joint value-distribution and universality of the Riemann zeta-function $\zeta(s)$ and the Hurwitz zeta-function $\zeta(s, \alpha)$ with transcendental parameter α were obtained by H. Mishou in 2007 (see [1]) while discrete versions were studied by E. Buivydas and A. Laurinčikas in 2015 (see [3], [2]).

In the talk, we will present the joint discrete mixed limit theorems and the joint discrete mixed universality theorems for the pair consisting of the L-function for new forms $L(s, F)$ and the periodic Hurwitz zeta-functions $\zeta(s, \alpha; \mathfrak{B})$. We will discuss on both cases of discreteness: with the same step of arithmetic progression h for both shifts $L(s + ikh, F)$ and $\zeta(s + ikh, \alpha; \mathfrak{B})$, and (more general) with different steps h_1 and h_2 for different shifts $L(s + ikh_1, F)$ and $\zeta(s + ikh_2, \alpha; \mathfrak{B})$.

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Gábor Kallós
(Széchenyi István University)

Beta expansions and dynamical systems

In this talk we will examine a possible interpretation of the β-expansions using dynamical systems. We will use the classic basic concepts and notations. So, let $\beta > 1$ be the (non-integer) base of a system, $k = [\beta]$. The general form of the fraction-expansion is:

$$x = \frac{\varepsilon_1}{\beta} + \frac{\varepsilon_2}{\beta^2} + \frac{\varepsilon_3}{\beta^3} + \cdots \tag{1}$$

where for the digits $\varepsilon_i \in \{0, 1, \ldots, k\}$ hold. All of the numbers x in $I_\beta = [0, k/(\beta - 1)]$ can be expressed this way.

As it is well-known (e.g. [4], [1]), the number of possible different expansions is in most of the cases continuum, but in many cases only countably infinite or finite. We can find even unique (univoque) expansions. The most interesting special expansions are the greedy (regular), the lazy and the quasi-regular one.

Let us define for $\beta \in (1, 2)$ the map $T_\beta : I_\beta \to I_\beta$ (greedy map) as follows [2]:

$$T_\beta(x) = \begin{cases} \beta x, & \text{if } 0 \le x < 1/\beta, \\ \beta x - 1, & \text{if } 1/\beta \le x \le 1/(\beta - 1). \end{cases}$$

Similarly, the lazy map $S_\beta : I_\beta \to I_\beta$ for $\beta \in (1, 2)$ is:

$$S_\beta(x) = \begin{cases} \beta x, & \text{if } 0 \le x \le 1/\beta(\beta - 1), \\ \beta x - 1, & \text{if } 1/\beta(\beta - 1) < x \le 1/(\beta - 1). \end{cases}$$

Under the maps, we can order itinerary and orbit(s) to the points. Is is clear that the itinerary of an $x \in I_\beta$ generates the greedy (regular) and the lazy expansions of x, using the greedy and lazy maps, respectively. These definitions can be extended simply to cover the cases $\beta > 2$.

We will present interesting examples to show the graphical behaviour of the possible orbits. The classic Erdős-Joó example (only two possible expansions for number 1) will also be considered [3].

In the analysis, we will focus on the very important question, what does it mean, if we have a choice in a given position? Finally, we will discuss the existence of continuum possible expansions.

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Tibor Kiss
(University of Debrecen)

On two functional equations related to equality problem of means

(joint work with Zsolt Páles)

The aim of the talk is to highlight important relationships between two functional equations related to the equality problem of arithmetic means and Cauchy means and of arithmetic means and Bajraktarević means, respectively.

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Oleg Klesov

(National Technical University of Ukraine)

Dual objects in function theory, probability theory and number theory

(joint work with Valerii V. Buldygin, Karl-Heinz Indlekofer and Josef G. Steinebach)

Let π_n be the number of prime numbers up to n and let p_n be the n^{th} prime number. Then

$$\pi_{p_n} = n. \tag{1}$$

Such a relation between two sequences is not rare in mathematics. Below is a similar example from the theory of probability. Let $\{X_n\}$ be a sequence of independent identically distributed random variables with a continuous distribution function. Put

$$M_n = \max\{X_1, \ldots, X_n\}, \qquad n \geq 1.$$

We say that n is a *record moment* if

$$X_n > M_{n-1}.$$

By definition, $n = 1$ is a record moment. The number of records up to the moment n is denoted by $N(n)$. If the moment when an n^{th} record occurs is denoted by $\tau(n)$, then

$$N(\tau(n)) = n. \tag{2}$$

Mathematical theory of records is a well developed branch of probability theory where some pioneering ideas and results are due to A. Rényi. Note that (2) is nothing else but (1) written by using a different notation.

Relation (2) recalls the defining properties of inverse functions in calculus

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x. \tag{3}$$

Note however that symbols π and p in (1) as well as N and τ in (2) are not interchangeable like f and f^{-1} in (3).

The aim of this talk is to study even more general objects, called *dual*, than in (1) or in (2). We establish limit properties of one of the dual objects by knowing the corresponding asymptotic behavior of another one.

The main question discussed is as follows: if f and g are dual objects and f is asymptotically equivalent to $f^{\sim}$, then what is the asymptotic behavior of g?

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Oleksiy Klurman
(Royal Institute of Technology, KTH)

Joint distribution of values of multiplicative functions

In this talk, we describe how one can combine recent breakthroughs by Matomäki, Radziwiłł and Tao, Szemerédi's theorem for long arithmetic progressions and some earlier work by the speaker to answer some open questions of Erdős, Kátai, Daróczy, Indlekofer and Phong. The talk is based on joint works with A. Mangerel.

Attila Kovács
(Eötvös Loránd University)

On lattice-based number systems

The generalization of positional number representations to various digit sets and to higher dimensions is a long story. Almost three hundred years ago (1727) the English mathematician Colson suggested to replace the digits $6, 7, 8, 9$ in the decimal system by $-1, -2, -3, -4$ in order to make the elementary calculations faster. Somewhat later, Cauchy advocated the same idea adding the digit -5 extra for getting even simpler calculations (1840). Grünwald (1885) investigated negative-based, Kempner (1936), Knuth (1960), Khmelnik (1964) and Penney (1965) complex-based systems. From the 70's Kátai, B. Kovács, Környei, Pethő (the "Hungarian school") and Gilbert examined systematically the radix extensions in *algebraic number fields*. In the 90's the *topological aspects* of radix representations was studied by Bandt, Indlekofer, Járai, Kátai, Lagarias, Wang, Vince, and later by Akiyama, Thuswaldner and others. The canonical number representation was generalized to *arbitrary polynomial systems* by Pethő (1989), and investigated later extensively by many authors (incl. Akiyama, Brunotte, A. Kovács, Pethő, Rao, Scheicher, Thuswaldner). The number system concept in *lattices* was investigated first by Vince (1993). The *algorithmical aspects* of canonical (polynomial) systems was initiated by Brunotte (2001) and for general lattices by A. Kovács. Recently, a special type of radix systems (SRS) is lengthly studied by Thuswaldner and his co-workers (the "Austrian school"). In this talk we overview the lattice-based number system concept for various aspects.

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Antanas Laurinčikas
(Vilnius University)

Joint value distribution theorems for the Riemann and Hurwitz zeta-functions

Let $\zeta(s)$ and $\zeta(s,\alpha)$, $s=\sigma+it$, $0<\alpha\leqslant 1$, denote the Riemann and Hurwitz zeta-functions, respectively. We consider the simultaneous approximation of a given pair of analytic functions by shifts $\zeta(s+i\varphi(k)), \zeta(s+i\varphi(k),\alpha)$, $k\in\mathbb{N}$, with parameter α for which the set $\{(\log p:\ p \text{ is prime}), (\log(m+\alpha):\ m\in\mathbb{N}_0)\}$ is linearly independent over the field of rational numbers. Here $\varphi(t)$ is a real-valued increasing function on $\left[k_0-\frac{1}{2},\infty\right)$ for a certain $k_0\in\mathbb{N}$, has a derivative $\varphi'(t)$ satisfying

$$\varphi(2t)\max_{t\leqslant u\leqslant 2t}\left(\varphi'(u)\right)^{-1}\ll t,$$

and such that the sequence $\{a\varphi(k):\ k\geqslant k_0\}$ with every $a\neq 0$ is uniformly distributed modulo 1. If the above conditions are satisfied, then, for every $\varepsilon>0$,

$$\liminf_{N\to\infty}\frac{1}{N-k_0+1}\#\left\{k_0\leqslant k\leqslant N: \sup_{s\in K_1}|\zeta(s+i\varphi(k))-f_1(s)|<\varepsilon,\right.$$
$$\left.\sup_{s\in K_2}|\zeta(s+i\varphi(k),\alpha)-f_2(s)|<\varepsilon\right\}>0,$$

where K_1 and K_2 are compact subsets of the strip $\left\{s\in\mathbb{C}:\frac{1}{2}<\sigma<1\right\}$ with connected complements, $f_1(s)$ is a continuous non-vanishing function on K_1 and analytic in K_1°, and $f_2(s)$ is a continuous function on K_2 and analytic in K_2°. The details can be found in [1].

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László Losonczi
(University of Debrecen)

On the zeros of reciprocal polynomials

Reciprocal polynomials are studied whose zeros are located in certain subsets of the complex plane. Of particular interest are the half planes $\Re z < 0$, $\Re z > 0$, the positive and negative half lines and the unit circle. Our main tool is the Chebyshev transform (see e.g. Lakatos [1]) and a Viéta–like formula for reciprocal polynomials (see Losonczi [2]). Using these we find necessary conditions, in some cases necessary and sufficient conditions for the reciprocal polynomials to have their zeros in the above sets.

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Levente Lócsi
(Eötvös Loránd University)

The Blaschke group and rational Zernike functions

(joint work with Zsolt Németh and Ferenc Schipp)

The Zernike polynomials form an orthogonal system on the unit disk. These functions are widely applied in medical engineering to describe optical behaviour of the human eye (the cornea). Utilizing the Blaschke functions and representations of the Blaschke group, we may transform the original Zernike basis to acquire new orthogonal systems on the disk [1], together with appropriate discretization [2, 3]. The main goal of the talk is to present a relation between the Blaschke group and Zernike functions [4], introducing matrix valued multiplicative characters. We will focus on the subgroup where one point of the unit circle remains fixed, thus this subgroup can be described using only one parameter on the unit disk. Furthermore the Zernike functions may be constructed by the translations of this group.

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Eugenijus Manstavičius
(Vilnius University)

Probabilistic number theory on permutations

The Turán-Kubilius inequality (T-KI) for additive number theoretic functions continues to be one of the main whales keeping on probabilistic number theory. The same role is playing the analogous result for additive functions defined on permutations taken at random from the symmetric group $\mathbb{S}_n$, $n \geq 1$. The latter parallel theory, being more simple, reveals all important points and obstacles to be overcome. For example, in this case, T-KI (with the exact constant in it!) follows from the fact that the spectrum of matrix

$$\Big(\Big(\frac{\mathbf{1}\{i+j>n\}}{\sqrt{ij}}\Big)\Big)_{1\leq i,j\leq n}$$

is $\{(-1)^{r-1}/r,\ 1 \leq r \leq n\}$. Here $\mathbf{1}\{\cdot\}$ is the indicator function.

Developing this, we intend to focus on the recently obtained moment inequalities when the symmetric group is equipped with a weighted probability measure.

Gábor Nagy

(Eötvös Loránd University)

On non-integer based expansions of real numbers in a special form

The research of non-integer based expansions of real numbers was started by A. Rényi and W. Parry ([3], [2]), a summary of the results of P. Erdős et al. in this field by G. Kallós can be find in [1]. The work of the aforementioned authors is about the expansions in the form:

$$x = \sum_{n=1}^{\infty} \varepsilon_n \lambda^n,$$

where $0 < \lambda < 1$ and $\varepsilon_n \in \{0, 1, \ldots, \lceil \frac{1}{\lambda} \rceil\}$.
I. Kátai proposed the investigation of the expansions in the form:

$$x = \sum_{n=1}^{\infty} \varepsilon_n \lambda^n + \delta_n \omega_n,$$

where ω_n is dependent on λ and n, one can choose values for ε_ns, but the values of δ_ns are determined by an opponent. This talk is about what numbers can be represented this way in the cases $\omega_n = a\lambda^n$ and $\omega_n = (a\lambda)^n$.

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Gergő Nagy
(University of Debrecen)

Transformations on positive operators with preserver properties related to means

(joint work with Marcell Gaál)

In this talk, maps on positive operators satisfying invariance equations containing so-called Kubo-Ando means are investigated. These means are defined via a system of four axioms. They have the important property that all of them are operations on the sets of all positive operators on a Hilbert space H and of all positive operators on H which are bounded above by the identity. Thus, a natural problem arises: describe the structure of homomorphisms of the latter sets with respect to a Kubo-Ando mean. It is important to remark that those transformations are exactly the maps which preserve the given mean in a certain sense. Another problem related to the above is to determine the general form of transformations on the mentioned sets leaving a norm of a Kubo-Ando mean invariant. In the talk, we present the solution of the previous problems in some cases.

Károly Nagy
(University of Nyíregyháza)

Cone-like restricted summability of the two-dimensional Walsh- and Walsh-Kaczmarz means

The properties of the maximal operators of the (C, α)-means ($\alpha = (\alpha_1, \dots, \alpha_d)$) of the multi-dimensional Walsh-Fourier and Walsh-Kaczmarz-Fourier series are discussed [3, 4, 5], where the sets of indices are inside a cone-like set [2]. The boundedness of the maximal operators from H_p^γ to L_p for $p_0 < p$ ($p_0 = \max 1/(1 + \alpha_k) : k = 1, \dots, d$) are treated. The endpoint case $p = p_0$ investigated, as well [1].

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Zsolt Németh
(Eötvös Loránd University)

Projection properties of de la Vallée Poussin type operators

Let us denote by $C_0(2\pi)$ the space of continuous 2π-periodic functions equipped with the maximum norm. Denoting the subspace of trigonometric polynomials of degree at most n by $\mathcal{T}_n$, the Fourier projection $F_n : C_0(2\pi) \to \mathcal{T}_n$ (for which $F_n f$, $(f \in C_0(2\pi))$ is the n-th partial sum of the trigonometric Fourier series of f) is a linear operator with the property $F_n|_{\mathcal{T}_n} \equiv$id. In fact, the Faber–Marcinkiewicz–Berman theorem states that F_n has the minimal norm among all projections from $C_0(2\pi)$ to $\mathcal{T}_n$ with the above property. This result stays true if we replace $C_0(2\pi)$ with the space of (Lebesgue) integrable functions $L_1(0, 2\pi)$.

Now, for natural numbers $m \le n$, we consider the sets of generalised projections $\{P \in \mathcal{L}(X, \mathcal{T}_n) : P|_{\mathcal{T}_m} \equiv \text{id}\}$, where $X = C_0(2\pi)$ or $X = L_1(0, 2\pi)$. Note that for $m := n$, we obtain the previously described operators. It turns out that in many cases, the minimal element of these sets can be described by arithmetic means of some Fourier projections, i.e. by some kind of (generalized) de la Vallée Poussin sum. In this talk, we investigate this relation between the aforementioned minimal projections and the corresponding de la Vallée Poussin sums.

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Vincent Ouellet
(University of Laval)

On the Middle Prime Factors of Integers

This talk will focus on new results regarding the asymptotic behavior of the sum of the reciprocals of the middle prime factors of an integer, and related matters. This will generalize and expand earlier work of De Koninck, Doyon, Luca and Ouellet.

Paweł Pasteczka
(Pedagogical University of Cracow)

Jensen-type geometric shapes

During *Conference on Inequalities and Applications 2016* Páles stated the problem whether for every convex and closed set X and every convex function $f\colon X \to \mathbb{R}$, the inequality

$$\frac{1}{|X|}\int_X f(x)\,dx \le \frac{1}{|\partial X|}\int_{\partial X} f(x)\,dx$$

is valid (∂X stands for the boundary of X).

In general the answer is negative, however we present some conditions to the convex closed shape X such that this inequality is true.

In particular if X is (i) an n-dimensional parallelotope, (ii) an n-dimensional ball, (iii) a convex polytope having an inscribed sphere (tangent to all its facets) with center in the center of mass of ∂X.

These results generalize the one of Dragomir and Pearce [1, Theorem 215], where such inequality was obtained for three dimensional ball.

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Zsolt Páles
(University of Debrecen)

Equality of Bajraktarević means with quasi-arithmetic means

(joint work with Amr Zakaria)

In this talk we characterize the situations when a two-variable symmetric Bajraktarević mean is necessarily a quasi-arithmetic mean. More precisely, we want to characterize those pairs of continuous functions $(f, g) : I \to \mathbb{R}^2$ for which there exists a continuous strictly increasing function $\varphi : I \to \mathbb{R}$ such that the functional equation

$$\left(\frac{f}{g}\right)^{-1}\left(\frac{f(x)+f(y)}{g(x)+g(y)}\right) = \varphi^{-1}\left(\frac{\varphi(x)+\varphi(y)}{2}\right) \qquad (x, y \in I)$$

holds.

Štefan Porubský
(Academy of Sciences of the Czech Republic)

Semigroup structure of sets of solutions to equation $X^m = X^s$

In our talk we generalize the results of [1] on the semigroup and group structure of the set of solutions to equation $X^m = X^s$ over multiplicative semigroups of factor rings of residually finite commutative rings and of factor rings of residually finite commutative principal ideal domains. Our approach makes use of the idempotent technique developed for periodic commutative semigroups and based on the properties of the maximal subsemigroups and maximal groups corresponding to an idempotent of the semigroup under consideration. In the case of residually finite principal ideal domains it employs the corresponding idempotent analysis of the Euler-Fermat Theorem as given in [2]. The special case when the set of solutions is a union of groups covers the sets of solutions to equations of the type $x^m = x$.

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Gábor Román
(Eötvös Loránd University)

On the expected number of curve orders during the Atkin-Morain primality test

Atkin and Morain described the background and an exact implementation of the elliptic curve primality proving algorithm in their [1] paper. The authors of [2] reduced the heuristic running time of this algorithm to $o(\ln^4 n)$, where n is the input number. They posed questions, for which the answers would strengthen their assumptions. One of the questions was about the asymptotic behaviour of the expected number of curve orders computed during a recursive stage of the test. The main result of [3] is the answer to this question.

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Lajos Rónyai
(Hungarian Academy of Sciences)

Recent results on norm-graphs

(joint work with Tomas Bayer, Tamás Mészáros and Tibor Szabó)

Let q be a prime power, $t > 1$ be an integer and denote by $N : \mathbb{F}_{q^{t-1}} \to \mathbb{F}_q$ the norm map from the finite field $\mathbb{F}_{q^{t-1}}$ to $\mathbb{F}_q$. The projective norm-graph $NG(q,t)$ was defined by Alon–Rónyai–Szabó [1]. The vertex set of $NG(q,t)$ is $\mathbb{F}_{q^{t-1}} \times \mathbb{F}_q^*$. Two vertices (A,a) and (B,b) of the graph are adjacent if and only if $N(A+B) = ab$. The projective norm graphs $NG(q,t)$ are known to provide tight constructions for the Turán number of complete bipartite graphs $K_{t,s}$ with $s > (t-1)!$, in particular $NG(q,4)$ does not contain $K_{4,7}$ subgraphs.

Here we prove that $NG(q,4)$ does contain (many copies of) $K_{4,6}$ for any prime power q not divisble by 2 or 3. We prove this by studying certain systems of norm equations over finite fields. Along the way we also count the copies of any fixed 3-degenerate subgraph, and find that projective norm graphs display quasirandom behaviour with respect to this parameter. Moreover, we determine the automorphism group of $NG(q,t)$.

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Maciej Sablik
(Silesian University of Katowice)

An elementary method of solving functional equations

In the present paper we prove in an elementary way that if for a fixed $\lambda \in (0,1)$ the functions $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = g\left(\lambda a + (1 - \lambda) b\right)$$

for all $b > a$ then f is a quadratic polynomial, and $g = f'$. Moreover, if $\lambda \neq \frac{1}{2}$, then f is a linear polynomial and $g = f'$. This result is obtained with no regularity assumptions on f or g and generalizes a theorem from [1].

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Peter Stadler
(University of Innsbruck)

The Short Ruler on the Real Projective Space

The restriction to the interval $[0, 1]$ of a homomorphism $h\colon (\mathbb{R}, +) \to (G, \circ)$ on a Lie group G is a geodesic. On Riemannian manifolds (M, g) geodesics are locally shortest lines. The problem is to construct long geodesics. We assume that we have a short ruler, which allows to construct geodesics with length $L > 0$. We can shorten a curve α on M using the short ruler (reduced transformation).

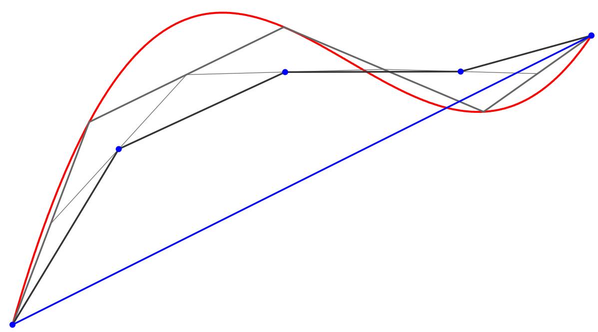

Figure 1. The reduced transformation.

The reduced process $(R_L^t \alpha)_{t\in\mathbb{N}}$ is the iteration of this transformation. In normed vector spaces, the reduced process converges to the straight line. On *complete* Riemannian manifolds, at least a subsequence converges to a geodesic γ. In the real projective space $\mathbb{R}P^m$ endowed with the standard round metric, the limit point is unique if γ is a non-closed or a simple closed geodesic.

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Gediminas Stepanauskas
(Vilnius University)

Asymptotical behaviour of arithmetic functions on shifted primes

(joint work with Jonas Šiaulys)

We will discus about the distributions of additive arithmetic functions and sums of such functions on shifted primes, i.e.

$$\frac{1}{\pi(x)}\#\{p < x : f(p+1) < u\},$$

$$\frac{1}{\pi(x)}\#\{p < x : f(p+1) + g(p+2) < u\},$$

as $x \to \infty$, and where f and g are additive functions.

The second part of our talk will be devoted to possible limit distributions of above distributions and to recent results concerning uniform distribution.

Kristóf Szarvas
(Eötvös Loránd University)

The boundedness of the Cesaro means in variable dyadic Hardy spaces

If $f \in L_p$ $(1 < p < \infty)$, then $\lim_{n\to\infty} s_n f = f$ in the L_p-norm, where $s_n f$ denotes the n-th partial sum of the Walsh-Fourier series (see e.g. Schipp, Wade, Simon and Pál [1]). Jiao, Zhou, Weisz and Wu [2] generalized this result for $L_{p(\cdot)}$: if $p(\cdot) \in \mathcal{P}(\Omega)$, $1 < p_- := \text{ess}\inf_{x\in\Omega} p(x) \leq p_+ := \text{ess}\sup_{x\in\Omega} < \infty$ and for all atoms A, the exponent function $p(\cdot)$ satisfies that

$$\mathbb{P}(A)^{p_-(A)-p_+(A)} \leq K_{p(\cdot)},$$

then for all $f \in L_{p(\cdot)}$, $\lim_{n\to\infty} s_n f = f$ in the $L_{p(\cdot)}$-norm. Unfortunately, these results are not true if $p \leq 1$ or if $p_- \leq 1$. Although, for $p \leq 1$, or $p_- \leq 1$, we can prove convergence results with the help of Cesaro means. For $\alpha > 0$ and $n \in \mathbb{N}$, the Cesaro means of the martingale f is defined by

$$\sigma_n^\alpha f := \frac{1}{A_{n-1}^\alpha} \sum_{k=1}^{n} A_{n-k}^{\alpha-1} s_k f = \frac{1}{A_{n-1}^\alpha} \sum_{k=0}^{n-1} A_{n-k-1}^{\alpha} \widehat{f}(k) w_k,$$

where A_k^α denotes the binomial coefficient $\binom{k+\alpha}{k}$.

We consider two types of maximal operators ($U^{(\alpha)}$ and $V^{(\alpha)}$) and we prove that (under some conditions) each maximal operator is bounded from the classical dyadic martingale Hardy space H_p to the classical Lebesgue space L_p and these maximal operators are bounded on $L_{p(\cdot)}$. Using these results, the boundedness of the Cesaro means from $H_{p(\cdot)}$ to $L_{p(\cdot)}$ can be proved. As a consequence we get results about almost everywhere and norm-convergence of the Cesaro means in $H_{p(\cdot)}$.

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Patrícia Szokol
(University of Debrecen)

Transformations preserving generalized quasi-arithmetic means of invertible positive operators

(joint work with Gergő Nagy)

In the paper [1], transformations on Hermitian matrices with invariance properties related to quasi-arithmetic means are investigated. These means are defined via the well-known formula used in the case of real numbers. They have the important property that all of them are operations on the sets of positive definite, semidefinite complex matrices of a given size. Thus, a natural problem arises: describe the structure of homomorphisms of the latter sets with respect to a quasi-arithmetic mean. It is important to remark that those transformations are exactly the maps which preserve the given mean in a certain sense. Another problem related to the above is to determine the general form of transformations on the mentioned sets leaving a norm of a quasi-arithmetic mean invariant. In the paper [1] Gaál and Nagy gave the solution of the previous problems in some cases.

In this talk we will answer the following question, which was raised by Professor Zoltán Daróczy: what can we state in the case of generalized quasi-arithmetic means? (The generalization of quasi-arithmetic means of real numbers are introduced in [2] by Professor Janusz Matkowski.)

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Rodolfo Toledo
(University of Nyíregyháza)

Numerical solution of linear differential equations by Walsh polynomials approach

In 1975 (see [1] and [2]) C. F. Chen and C. H. Hsiao established a new procedure to solve initial value problems of systems of linear differential equations with constant coefficients by Walsh polynomials approach. The basic idea is to avoid differentiation considering the equivalent integral equations instead the original differential equations. In that way, the substitution of the partial sums of Walsh-series in the integral equation reduces the problem to solve a linear system. However, they did not deal with the analysis of the proposed numerical solution.

In joint work with György Gát (see [3]) I studied this procedure in case of one equation with the techniques that the theory of dyadic harmonic analysis provides us. I refer to determine if the linear system from which we obtain the coefficients of the Walsh polynomials is solvable or not and also to the estimation of errors. Moreover, we proposed a faster multistep method to obtain directly the values of the numerical solution without needing to generate Walsh functions or to solve linear systems.

We have now been able to establish a similar method to solve initial value problems of differential equations with not necessarily constant coefficients. In my talk I show the main results concerning the analysis of this method and I illustrate it through some examples.

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Pavel Varbanets

(I.I. Mechnikov Odessa National University)

Exponential sums over the sequences of PRN's produced by inversive generators

(joint work with Sergey Varbanets)

The inversive congruential method for generating uniform pseudorandom numbers is a particulary attractive alternative to linear congruential generators, which show many undesirable regularities. The character of equidistribution the sequence of pseudorandom numbers (abbreviate, PRN's) is defined by the discrepancy of this sequence. Usually the bound of discrepancy for the sequence of PRN's, that is generated by the congruential generator, is estimated by using the Turan-Erdös-Koksma inequality, the core of which is the exponential sum with elements of this sequence in exponent. The exponential sums on inversive congruential pseudorandom numbers are estimated and for which the following results obtained.

Theorem. *Let $\{y_n\}$ be the sequence of PRN's produced by recursion (ℓ), $\ell = I, II$, where a is a quadratic non-residue modulo p. Then,*

$$S_\ell^{(j)}(h;N) = \sum_{\substack{n=0 \\ y_n \in (\ell)}}^{N-1} e^{2\pi i \frac{h y_n^j}{p^m}} \ll p^{\frac{m+\nu_0}{2}}, \;\; j \in \mathbb{Z}, \;\; (j,p)=1, \;\; (h,p)=1;$$

hold.

Theorem. *Let h_1, h_2 be arbitrary integers with $s = \nu_p(gcd(h_1,h_2,p^m))$, $s \le m - \nu_0$. Then for the sequence $\{y_n\}$ produced by recursion (ℓ), $\ell = I, II$ and with a maximal period length $\tau = 3p^{m-\nu_0}$ we have*

$$K_\ell(h_1,h_2;N) = \sum_{\substack{n=0 \\ y_n \in (\ell)}}^{N-1} e^{2\pi i \frac{h_1 y_n + h_2 y_n^{-1}}{p^m}} \ll p^{\frac{m+\nu_0+s}{2}},$$

Peter Volkmann
(Karlsruhe Institute of Technology)

An application of a comparison theorem for functional equations

(joint work with Gerd Herzog)

We like to discuss an application of a comparison theorem for functional equations in ordered topological vector spaces. For this purpose we use some operators which are inspired by the definition of Jensen convexity.

Ferenc Weisz
(Eötvös Loránd University)

Variable Hardy spaces and applications in Fourier analysis

Let $p(\cdot) : \mathbb{R}^n \to (0, \infty)$ be a variable exponent function satisfying the globally log-Hölder condition. We introduce the variable Lebesgue and Hardy spaces $L_{p(\cdot)}(\mathbb{R}^d)$ and $H_{p(\cdot)}(\mathbb{R}^d)$. A general summability method, the so called θ-summability is considered for multi-dimensional Fourier transforms. Under some conditions on θ, it is proved that the maximal operator of the θ-means is bounded from $H_{p(\cdot)}(\mathbb{R}^d)$ to $L_{p(\cdot)}(\mathbb{R}^d)$. This implies some norm and almost everywhere convergence results for the θ-means, amongst others the generalization of the well known Lebesgue's theorem. Some special cases of the θ-summation are considered, such as the Riesz, Bochner-Riesz, Weierstrass, Picard and Bessel summations.

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Invariance of a symmetric Bajraktarević mean with respect to two nonsymmetric Bajraktarević means

(joint work with Zsolt Páles)

Given two continuous functions $f, g : I \to \mathbb{R}$ such that g is nowhere zero on I and the ratio function f/g is strictly monotone on I, the *weighted two-variable Bajraktarević mean* $B_{f,g} : I^2 \times \mathbb{R}_+^2 \to I$ is defined by

$$B_{f,g}(x, y; t, s) := \left(\frac{f}{g}\right)^{-1}\left(\frac{tf(x) + sf(y)}{tg(x) + sg(y)}\right) \qquad (x, y \in I;\ s, t \in \mathbb{R}_+).$$

The purpose of this talk is to solve the invariance of a two-variable symmetric Bajraktarević mean with respect to two-variable weighted nonsymmetric Bajraktarević means, i.e., to solve the functional equation

$$\frac{\ell(B_{f,g}(x, y; t, s)) + \ell(B_{h,k}(x, y; s, t))}{m(B_{f,g}(x, y; t, s)) + m(B_{h,k}(x, y; s, t))} = \frac{\ell(x) + \ell(y)}{m(x) + m(y)} \qquad (x, y \in I),$$

where $f, g, h, k, \ell, m : I \to \mathbb{R}$ are unknown continuous functions such that g, k, m are nowhere zero on I, the ratio functions f/g, h/k, ℓ/m are strictly monotone on I, and $s, t \in \mathbb{R}_+$ are fixed different positive numbers. This equation is more general than the functional equation investigated in [3].

References

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